Solving Trigonometric Moment Problems for Fast Transient Imaging

Christoph Peters[1], Jonathan Klein[2,1], Matthias B. Hullin[1], Reinhard Klein[1]
[1]University of Bonn [2]French-German Research Institute of Saint-Louis

Introduction

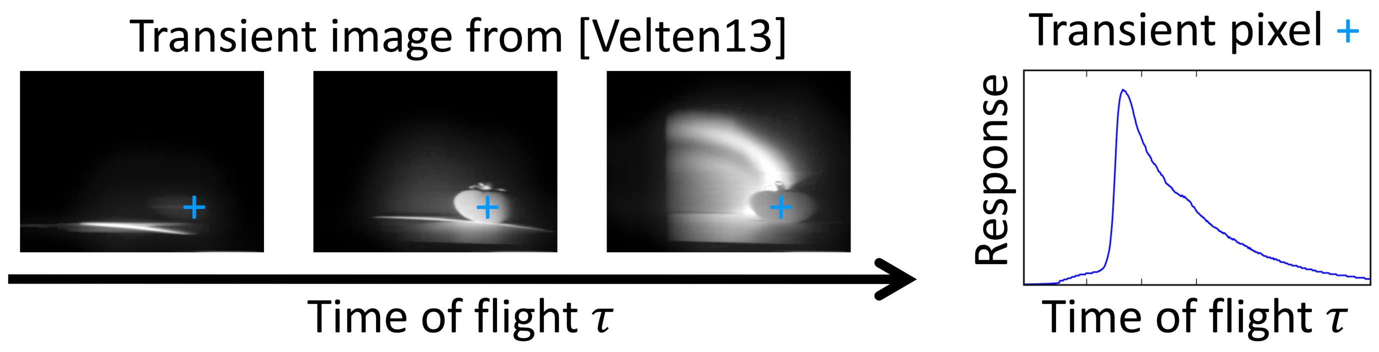
We speed up transient imaging drastically in terms of capture and reconstruction. This also enables improved range imaging.

Technique	Capture time	Hardware type	Hardware cost
[Velten13]	ca. 1 h	Streak camera	ca. 500,000 USD
[Heide13]	ca. 1 min	AMCW lidar	ca. 1000 USD
Ours [Peters15]	54 ms to 8.2 s	AMCW lidar	ca. 1000 USD

This work is published as SIGGRAPH Asia 2015 technical paper [Peters15].

Transient Imaging and AMCW Lidar

Per pixel a transient image provides a time-resolved impulse response. It shows how much light returns after a given time of flight.

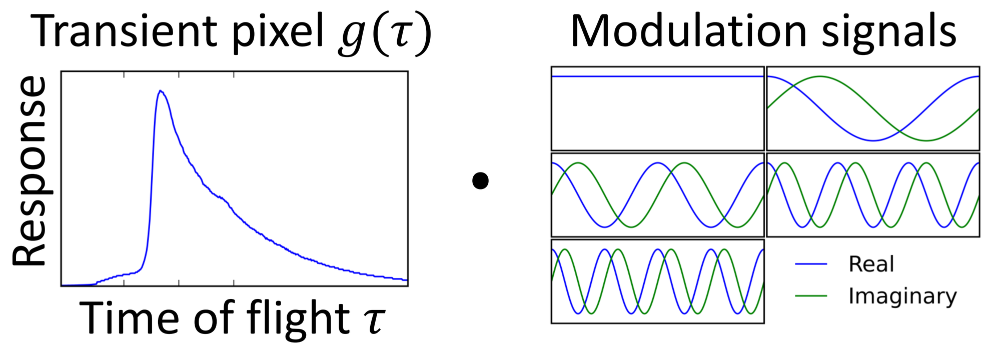


AMCW lidar correlates this impulse response with a periodic modulation.

Trigonometric Moments

Our AMCW lidar measurements use the Fourier basis for modulation. They are called trigonometric moments. We use $m \in \mathbb{N}$ multiples of a base frequency f (e.g. f=23 MHz) and frequency zero (no modulation).

$$\int_0^\infty \text{Transient pixel } g(\tau) \cdot \text{Modulation signals} \, d\tau$$

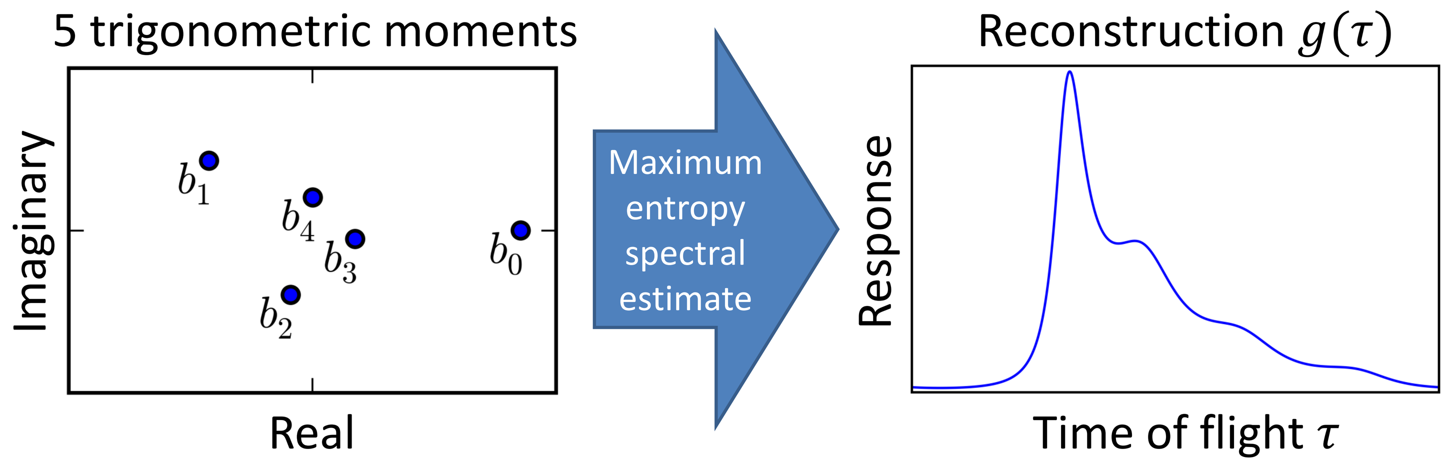


$$\forall j \in \{0, \ldots, m\}: \; b_j := \int_0^\infty g(\tau) \cdot \exp(j \cdot 2\pi i f \cdot \tau) \, d\tau$$

Reconstructing Continuous Responses

Reconstruction uses the maximum entropy spectral estimate [Burg75]:

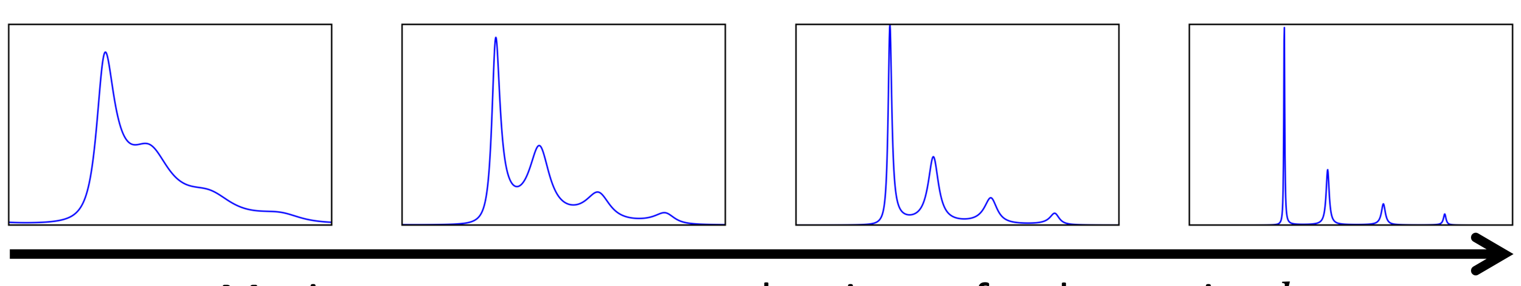
$$g(\tau) \propto \left| \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}^{\mathsf{T}} \cdot \begin{pmatrix} b_0 & \overline{b_1} & \cdots & \overline{b_m} \\ b_1 & b_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \overline{b_1} \\ b_m & \cdots & b_1 & b_0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} \exp(0 \cdot 2\pi i f \cdot \tau) \\ \exp(1 \cdot 2\pi i f \cdot \tau) \\ \vdots \\ \exp(m \cdot 2\pi i f \cdot \tau) \end{pmatrix} \right|^{-2}$$



- All measurements (i.e. moments) are used as hard constraint,
- The reconstruction is always positive and smooth.

Reconstructing Sparse Responses

As the ground truth approaches a sparse response with m peaks, the maximum entropy spectral estimate converges to a sparse response.



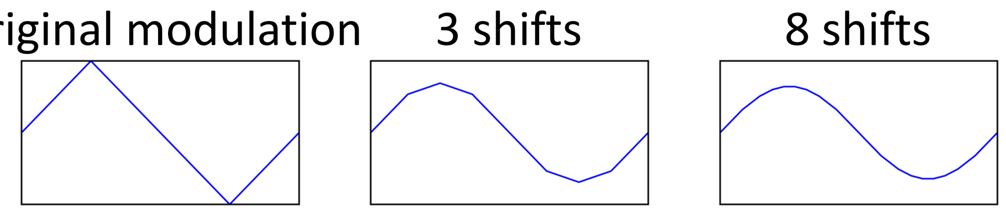
Maximum entropy spectral estimate for decreasing b_0

In the limit case, the so called Pisarenko estimate recovers it perfectly.

Measurement Procedure

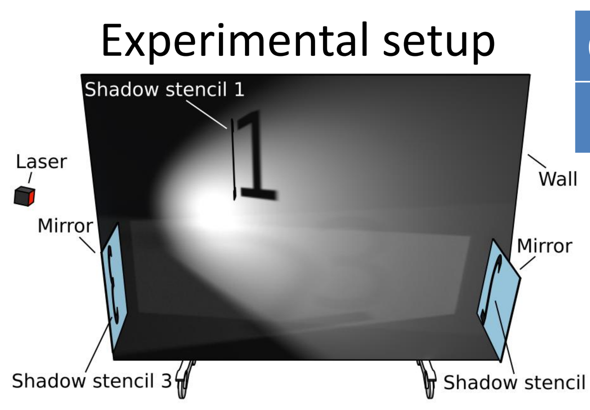
Our hardware is a CamBoard nano by PMDTechnologies (163·120 pixels) with custom laser illumination, modulation source and optics [Heide13].

Modulation is shifted 8 times during one exposure.

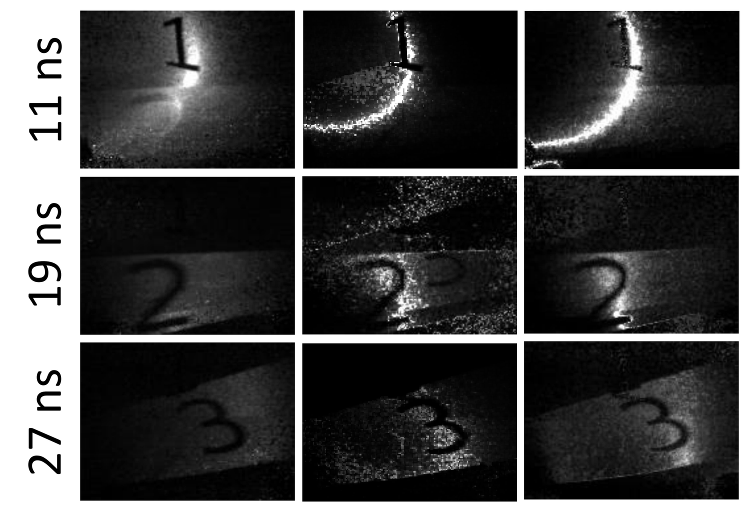


Transient Imaging

Experimental setup



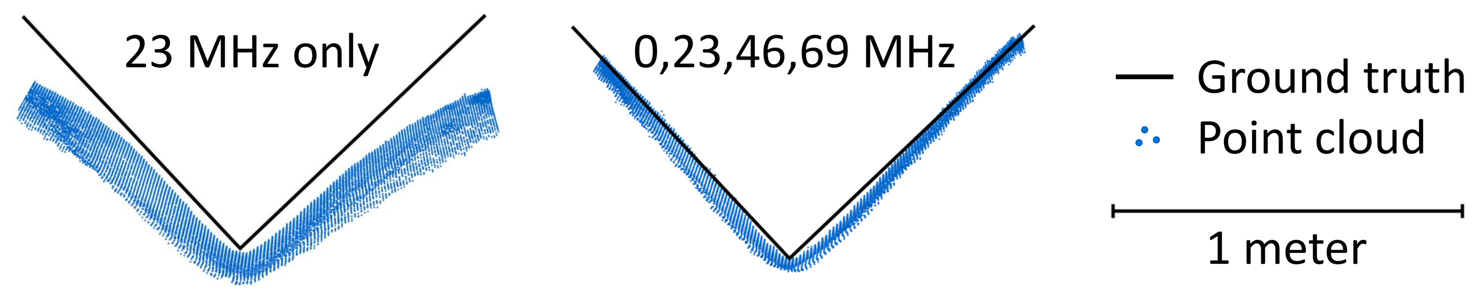
Capture time	90 ms	113 ms	8.2 s
m	3	4	8



We separate 3 responses with 3 frequencies. Results sharpen as the number of frequencies m increases. Longer exposure reduces noise.

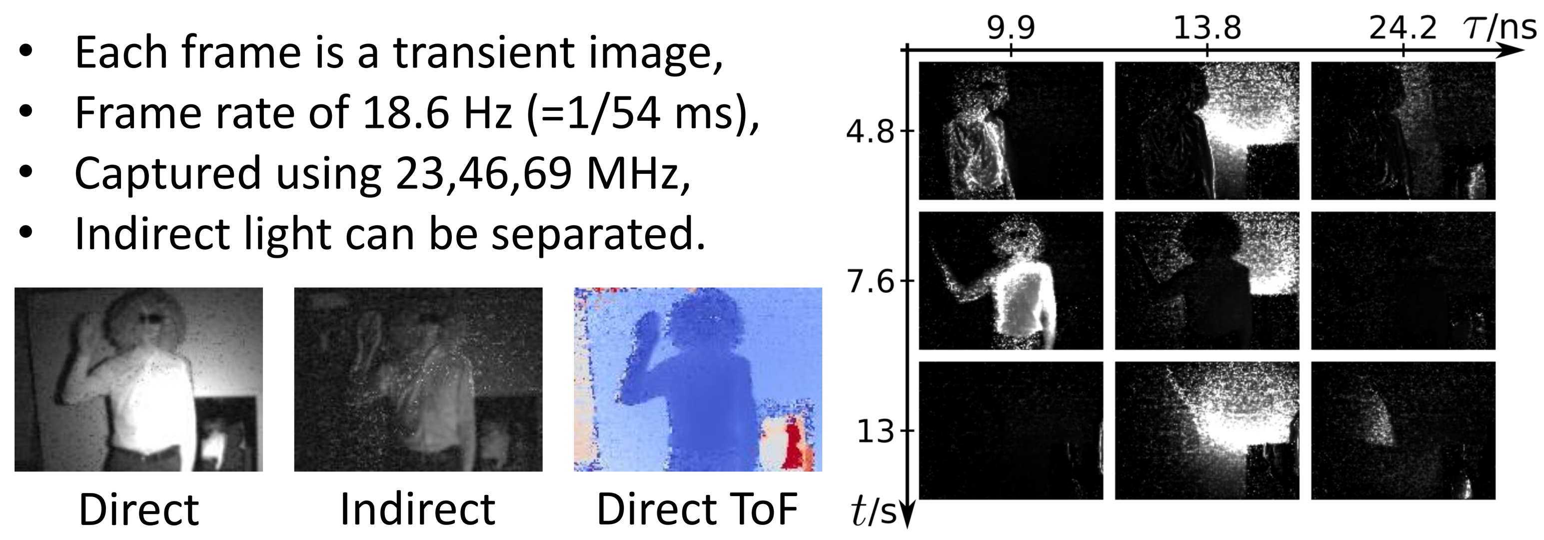
Range Imaging

Range estimate is local maximum of maximum entropy spectral estimate.



Transient Video

- Each frame is a transient image,
- Frame rate of 18.6 Hz (=1/54 ms),
- Captured using 23,46,69 MHz,
- Indirect light can be separated.



Conclusions

We are hoping that our work will soon lead to fast transient imaging and improved range imaging with inexpensive off-the-shelf hardware.

PMDTechnologies pico flexx Microsoft Kinect for Xbox One

Acknowledgments

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References

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[Burg75] J. P. Burg, Maximum Entropy Spectral Analysis, Ph.D. dissertation, Stanford University, Department of Geophysics, 1975
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